Some Results on Fixed Points of the Burrows-Wheeler Transform

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Cyclic Conjugates and Sorting

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A *run* of w is a maximal factor a^k with $a \in \mathcal{A}$, $k \geq 1$. We denote the number of runs in w by $\rho(w)$.

$$\mathrm{bwt}_{\mathcal{I}}(\text{vesuvio}) = \texttt{vv} \mid \texttt{i} \mid \texttt{e} \mid \texttt{s} \mid \texttt{o} \mid \texttt{u}, \qquad \rho = 6, \qquad |\mathrm{alph}(\mathrm{bwt}_{\mathcal{I}}(w))| = 6.$$

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A word w is *π-clustering* if $\mathrm{bwt}_{\mathcal{A}}(w) = a_{\pi(a_1)}^{k_1} a_{\pi(a_2)}^{k_2} \cdots a_{\pi(a_n)}^{k_n}$. The word w = vesuvio clusters with $\pi = (6, 2, 1, 4, 3, 5)$ when considered as a word over $\mathcal{A} = \{e < i < o < s < u < v\}$.

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Theorem (Mantaci, Restivo, Rosone, Sciortino, Versari 2017)

$$|\mathrm{alph}(w)| \leq \rho(\mathrm{bwt}_{\mathcal{A}}(w)) \leq 2\rho(w)$$

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The BWT is not surjective (e.g., $w = \mathrm{ab}$). A sequence of letters $(b_1, b_2, \ldots, b_n)$ has a *global ascent* if

$$\max_{1 \le i \le h} b_i \;\le\; \min_{h < j \le n} b_j \qquad \text{for some } 1 \le h < n.$$

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$b_1^{k_1} b_2^{k_2} \cdots b_n^{k_n}$, for some $k_i > 0$, $b_i \neq b_{i+1}$, is an image of the BWT if and only if $(b_1, \ldots, b_n)$ has no global ascent.

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The word $\mathrm{bwt}_{\mathcal{I}}(\text{vesuvio}) = \texttt{vviesou}$ has run sequence $(\texttt{v}, \texttt{i}, \texttt{e}, \texttt{s}, \texttt{o}, \texttt{u})$, and is thus the image of some word under the BWT. The word `eiosuvv`, however, is not a BWT image of any word.

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Corollary

If an image of the BWT is a Lyndon word, then it is a single letter.

Fixed Points

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A word $w \in \mathcal{A}^*$ is a *fixed point* of the BWT if $\mathrm{bwt}_{\mathcal{A}}(w) = w$. Sorting the conjugates of $w = \mathtt{pino}$ over $\mathcal{I} = \{i < n < o < p\}$ gives

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The Plan (for w with $|\mathrm{alph}(w)| > 2$)

$\Delta(w) = 0$	$\Delta(w) = 1$	$\Delta(w) \geq 2$
characterized	characterized	open

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A word w is *primitive* if $w = u^p$ implies $p = 1$. We have

$$\mathrm{bwt}_{\mathcal{A}}(u) = b_1 b_2 \cdots b_n \quad \Longrightarrow \quad \mathrm{bwt}_{\mathcal{A}}(u^p) = b_1^p b_2^p \cdots b_n^p.$$

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Proposition (Dolce, H. 2026+)

If $w \in \mathcal{A}^+$ is a fixed point of the BWT and $w = u^p$ with $p > 1$, then either $w = \mathtt{a}^p$ for a single letter $\mathtt{a}$, or $w = (\mathtt{b}^2\mathtt{a}^2)^2$ for $\mathtt{a} < \mathtt{b}$.

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In particular, $|\mathrm{alph}(w)| > 2$ implies w is primitive, so every fixed point onward is primitive.

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Theorem (Dolce, H. 2026+)

Let w be a word with $|\mathrm{alph}(w)| > 2$. Then w is a fixed point of the BWT with $\Delta(w) = 0$ if and only if $w = a_m a_1^{k_1} a_2^{k_2} \cdots a_{m-1}^{k_{m-1}}$, for some $a_1 < a_2 < \cdots < a_m$ in $\mathcal{A}$ and integers $k_i > 0$.

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The largest letter occurs once, followed by other smaller letters in increasing order. For instance, $w = \mathtt{eaabcdd}$ over $\{\mathtt{a} < \mathtt{b} < \mathtt{c} < \mathtt{d} < \mathtt{e}\}$.

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Note that every clustering fixed point is maximal in its conjugacy class.

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We now consider the case when exactly one letter has two runs. Over $\mathcal{A} = \{\mathtt{a} < \mathtt{b} < \mathtt{c}\}$ a fixed point with $\Delta(w) = 1$ has four runs.

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A word $w \in \{\mathtt{a} < \mathtt{b} < \mathtt{c}\}^*$ is a fixed point of the BWT with $\Delta(w) = 1$ if and only if

$$w = \mathtt{c}^{2k+1}\mathtt{a}\,\mathtt{c}^{k}\mathtt{b}, \qquad w = \mathtt{c}^{2k}\mathtt{b}\,\mathtt{c}^{k}\mathtt{a}, \qquad \text{or} \qquad w = \mathtt{c}^{2}\mathtt{b}^{k}\mathtt{c}\,\mathtt{a},$$

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The word $\mathtt{c}^2\mathtt{b}^k\mathtt{c}\,\mathtt{a}$ has an immediate analogue on larger alphabets.

$\Delta = 1$: *Four or More Letters*

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Theorem (Dolce, H. 2026+)

Let w be a fixed point of the BWT with $|\mathrm{alph}(w)| > 3$ and $\Delta(w) = 1$. Then $w = a_m^2 a_2^{k_2} a_3^{k_3} \cdots a_{m-1}^{k_{m-1}} a_m a_1$ for some $a_1 < a_2 < \ldots < a_m \in \mathcal{A}$ and positive integers k_i.

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Examples :

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- ddbccda
- ddbcccccccccccccccccccccccccccccccccccccccccda

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Proposition

The word $w = a_{m-1}^2 a_1^2 a_m a_2^+ a_3^+ \cdots a_{m-2}^+ a_{m-1} a_1^2$ over $\mathcal{A} = \{a_1 < \ldots < a_m\}$ is a fixed point of the BWT when $m \geq 3$.

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We do not have a full list of families for any $\Delta \geq 2$.

The BWT and IET Languages

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Corollary (Dolce, H. 2026+)

Let T be an IET over the alphabet $\mathcal{A} = \{a_1 < \ldots < a_n\}$. If there exists a pangrammatic fixed point of the BWT in the set of return words in $\mathcal{L}(T)$, then the permutation associated with T is $\pi = (a_1\ a_n\ a_{n-1} \ldots\ a_2)$.

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Corollary (Dolce, H. 2026+)

Let T be a symmetric IET over the alphabet $\mathcal{A} = \{a_1 < \ldots < a_n\}$. Every fixed point in the set of return words in $\mathcal{L}(T)$ contains at most two letters.

Grazie mille per l'attenzione !