Test Two Supplementary Practice Test

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BIE-LA1 - Winter 2025

1. Let
$$M = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 3 & 4 \\ 2 & 0 & 1 & 5 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$
with entries in $\mathbb{Z}_7$. Is M invertible? If so, compute its inverse.

2. Consider the lists of vectors $L_1 = \left(\begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}\right)$ and $L_2 = \left(\begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}\right)$ of vectors in $\mathbb{R}^4$.
   (a) Is it true that $\text{span}L_1 = \text{span}L_2$?
   (b) Is L_1 a basis for $\mathbb{R}^4$? If not, extend it to a basis for $\mathbb{R}^4$.
   (c) Denote the basis of part (b) by $\mathcal{B}$. Compute $[I]_{\mathcal{E}\leftarrow\mathcal{B}}$.

3. Consider the basis $\mathcal{B} = \left(\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}\right)$.
   (a) Compute $[I]_{\mathcal{E}\leftarrow\mathcal{B}}$ and $[I]_{\mathcal{B}\leftarrow\mathcal{E}}$.
   (b) Let $\mathcal{B}'$ be the basis $\left(\begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 6 \\ -2 \\ 6 \end{bmatrix}\right)$. Compute $[I]_{\mathcal{B}'\leftarrow\mathcal{B}}$ and $[I]_{\mathcal{B}\leftarrow\mathcal{B}'}$.

4. Let
$$M = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 3 & 4 \\ 1 & -1 & 2 & 0 \end{pmatrix}.$$
   (a) Compute $\ker(M)$ and $\text{ran}(M)$.
   (b) What is the dimension of $\ker M$ and what is the rank of M? Justify your response.
   (c) Describe the range of M^{T}. What is the rank of M^{T}?

5. (Disclaimer: according to updated information, you are unlikely to have true/false questions on your second test. I am leaving these here because I think they are good practice.)

(a) **True/False:** There is a spanning list of length 3 in $\mathbb{Z}_{13}^2$.

(b) **True/False:** There is a linearly independent list of length 7 in $\mathbb{R}^6$.

(c) **True/False:** There is a matrix $A \in M_{5,7}(\mathbb{R})$ with $\dim(\ker A) = 2$ and $\text{rank}(A) = 4$.

(d) **True/False:** Each matrix $A \in M_{8,8}(\mathbb{R})$ with $\text{rank}(A) = 0$ has linearly independent columns.